**Equations of First Order and First Degree**

**Definition:**

A differential equation of the type, where and are functions of  andor constants is called a differential equation of the first order and first degree.

There are two standard forms of differential equations of first order and first degree namely

* 
* 

We can classify the first order and first degree differential equation intofollowings eight categories according to its solution methods:

* Equations of variable separable form,
* Equations reducible to variable separable form,
* Homogeneous Equation,
* Equation reducible to Homogeneous form,
* Linear differential equation,
* Equation reducible to linear differential equation,
* Exact differential equation and
* Equation reducible to exact differential equation.

**Equations of variable Separable form:**If an equation can be written in such a way that and all the term containing *x* are on one side and and all the term containing *y* are on other side, then this an equation in which variables are separable.



This type of equation can be solved by integrating directly and adding a constant on either side.

**Problem-01:**Solve the differential equation 

**Solution:**Given differential equation is,



Separating the variables, we get





Integrating both-sides, we find











which is the complete or general solution of the given differential equation.

**Problem-02:** Solve the differential equation.

**Solution:**Given differential equation is,



Separating the variables, we get



Integrating both-sides, we find

























which is the complete or general solution of the given differentialequation.

**Problem-03:** Solve the differential equation .

**Solution:**Given differential equation is,



Separating variables, we get



Integrating both-sides we find,







which is the complete or general solution of the given differential equation.

**Problem-04:**Solve the differential equation .

**Solution:**Given differential equation is,



Separating the variables, we get





Integrating both-sides, we find













which is the complete or general solution of the given differential equation.

**Problem-05:**Solve the differential equation .

**Solution:**Given differential equation is,



Separating the variables, we get







Integrating both-sides, we find







which is the complete or general solution of the given differential equation.

**Problem-06:**Solve the differential equation .

**Solution:**Given differential equation is,



Separating the variables, we get







Integrating both-sides, we find













which is the complete or general solution of the given differential equation.

**Problem 8:** Solve****

**Solution:** Given that,

****

Separating variables we obtain,





Now, integrating,











**(As desired)**

**Exercise:**

**Solve the following differential equations:**

1. 
2. 
3. 
4. 
5. 
6. 
7. 

**Equations reducible to variable separable form:**

* An equation of the form can be reduced to variable separable form by choosing the transformation.

**Problem-01:**Solve the differential equation .

**Solution:**Given differential equation is,



Let, 



Now from equation (1), we get







Integrating both sides, we find















which is the complete integral or general solution of the given differential equation.

**Problem-02:** Solve the differential equation .

**Solution:** Given differential equation is,



Let, 



Now from equation (1), we get







Integrating both sides, we find













which is the complete integral or general solution of the given differential equation.

**Problem-03:**Solve the differential equation .

**Solution:**Given differential equation is,



Let, 



Now from equation (1), we get







Integrating both sides, we get













which is the complete integral or general solution of the given differential equation.

**Problem-04:** Solve the differential equation .

**Solution:** Given differential equation is,

 … … … (1)

Let, 



Now from equation (1),we get













Integrating both sides, we get







which is the complete integral or general solution of the given differential equation.

**Problem-05:**Solve the differential equation .

**Solution:** Given differential equation is,

… … … (1)

Let, 



Now from equation (1), we get







Integrating both sides, we get

















which is the complete integral or general solution of the given differential equation.

**Problem-06:**Solve the differential equation .

**Solution:** Given differential equation is,





Let, 



Now from equation (1), we get











Integrating both sides, we get











which is the complete integral or general solution of the given differential equation.

**Exercise:**

**Solve the following differential equations:**

1. 
2. 
3. ****
4. ****
5. ****

**Homogeneous Differential Equation:** An equation of the form



In which  and  are homogeneous functions of  and  of the same degree is called homogeneous equation.

It can be reduced to an equation in which variables are separable by choosing

.

**Problem-01:**Solve the differential equation .

**Solution:** Given differential equation is,

... … … (1)

Equation (1) can be written as,





 … … … (2)

This is a homogeneous equation.

Put, 



From equation (2) we get













Integrating both sides, we get

















which is the complete integral or general solution of the given differential equation.

**Problem-02:**Solve the differential equation .

**Solution:** Given differential equation is,

 … ... … (1)

Equation (1) can be written as,





 … … … (2)

This is a homogeneous equation.

Let, 



From equation (2), we get













Integrating both sides, we get













which is the complete integral or general solution of the given differential equation.

**Problem-03:**Solve the differential equation .

**Solution:** Given differential equation is,

 … ... … (1)

Equation (1) can be written as,



… … … (2)

This is a homogeneous equation.

Let, 



From equation (2), we get











Integrating both sides, we get













which is the complete integral or general solution of the given differential equation.

**Problem-04:**Solve the differential equation .

**Solution:** Given differential equation is,

 … … … (1)

Equation (1) can be written as,



… … … (2)

This is a homogeneous equation.

Let, 



From equation (2), we get











Integrating both sides, we get

















which is the complete integral or general solution of the given differential equation.

**Problem-05:**Solve the differential equation .

**Solution:** Given differential equation is,

… … … (1)

Equation (1) can be written as,





… … … (2)

This is a homogeneous equation.

Let, 



From equation (2), we get













Integrating both sides, we get





















which is the complete integral or general solution of the given differential equation.

**Problem-06:**Solve the differential equation

.

**Solution:** Given differential equation is,

 … … … (1)

This is a homogeneous equation.

Let, 



From equation (1), we get













Integrating both sides, we get













which is the complete integral or general solution of the given differential equation.

**Exercise: Solve the following differential equations:**

1. ****
2. ****
3. ****
4. 
5. 

**Equation Reducible to Homogeneous Form:**An equation of the type

… … … (1)

can be reduced to homogeneous form as follows:

* Case -01: Ifthen putting  , and in equation (1) we get



we choose the constants  and  in such a way that,

and

with this substitution the differential equation reduces to



which is a homogeneous equation in , and can be solved by putting .

* Case-02: If (say), then the differential equation can be written as,



put so that 

the above equation becomes



which is in variables separable form.

**Problem-01:** Solve the differential equation 

**Solution:** Given differential equation is,

… …. … (1)

put and  where ,  are constants.



Then the equation (1) becomes,

… … … (2)

Now choose

… … … (3)

… … … (4)

Solving equations (3) and (4) we get,

and

with this substitution equation (2) becomes,

… … … (5)

which is a homogeneous equation in and .

So put, 



From equation (5), we have













Integrating both sides, we get

















which is the required solution.

**Problem-02:** Solve the differential equation 

**Solution:** Given differential equation is,



 … …. … (1)

put and  where ,  are constants.



Then the equation (1) becomes,

… … … (2)

Now choose

… … … (3)

… … … (4)

Solving equations (3) and (4) we get,

and

with this substitution equation (2) becomes,

… … … (5)

which is a homogeneous equation in and .

So put, 



From equation (5), we have















Integrating both sides, we get























which is the required solution.

**Problem-03:** Solve the differential equation 

**Solution:** Given differential equation is,





 … …. … (1)

put 



Then the equation (1) becomes,











Integrating both sides, we get













which is the required solution.

**Problem-04:** Solve the differential equation 

**Solution:** Given differential equation is,





 … …. … (1)

put 



Then the equation (1) becomes,













Integrating both sides, we get











which is the required solution.

**Exercise:** Solve the following problems:

1. 
2. 
3. 
4. .

**Linear Differential Equation:** A differential equation of the form



whereandare functions of  or constants, is called the linear differential equation of the first order.

**Note:** To solve this equation, multiply both sides by the following integrating factor.



**Problem-01:**Solve the differential equation.

**Solution:** Given differential equation is,

… … … (1)

Equation (1) can be written as,



… … … (2)

This is a linear equation of first order.

I.F

Multiply both sides of equation (2) by , we get







Integrating both sides, we get





which is the required solution.

**Problem-02:** Solve the differential equation.

**Solution:** Given differential equation is,

… … … (1)

Equation (1) can be written as,



… … … (2)

This is a linear equation of first order.

I.F

Multiply both sides of equation (2) by , we get





Integrating both sides, we get















which is the required solution.

**Problem-03:** Solve the differential equation.

**Solution:** Given differential equation is,

 … … … (1)

This is a linear equation of first order.

I.F

Multiply both sides of equation (1) by , we get





Integrating both sides, we get









which is the required solution.

**Problem-04:** Solve the differential equation.

**Solution:** Given differential equation is,

… … … (1)

The equation (1) can be written as,



… … … (2)

This is a linear equation of first order.

I.F

Multiply both sides of equation (2) by , we get





Integrating both sides, we get













which is the required solution.

**Problem-05:** Solve the differential equation.

**Solution:** Given differential equation is,

 … … … (1)

This is a linear equation of first order.

I.F

Multiply both sides of equation (1) by  , we get







Integrating both sides, we get







which is the required solution.

**Problem-06:** Solve the differential equation.

**Solution:** Given differential equation is,

… … … (1)

The equation (1) can be written as,









 … … … (2)

This is a linear equation of first order.

I.F

Multiply both sides of equation (2) by, we get





Integrating both sides, we get











which is the required solution.

**Problem-07:** Solve the differential equation.

**Solution:** Given differential equation is,

 … … … (1)

The equation (1) can be written as,





… … … (2)

This is a linear equation of first order.

I.F

Multiply both sides of equation (2) by, we get





Integrating both sides, we get







which is the required solution.

**Exercise:** Solve the following differential equations:

1. ****
2. ****
3. ****
4. ****
5. ****
6. ****

**Equations reducible to linear form: Bernoulli Equation:** An equation of the form

****

whereand are functions of or constants is called a Bernoulli Equation of first order.

**Theorem:** Reduce the Bernoulli Equation to Linear form and then solve it.

Answer: The Bernoulli’s equation is,

 … … … (1)

Dividing the equation (1) by we get

 … … … (2)

put, 



Now equation (2) transforms into,



… … … (3)

Let and then equation (3) becomes,

… … … (4)

which is a linear form.

**2nd part:** The integrating factor is,

*I.F*

Multiply both sides of equation (4) by we get,



****

Integrating both sides, we get



which is the required solution.

**Problem-01:** Solve the differential equation .

**Solution:** The differential equation is,

 … … … (1)

Equation (1) can be written as,



… … … (2)

This is a Bernoulli’s equation.

Dividing the equation (2) by we get

… … … (3)

put



Now the equation (3) becomes,



… … … (4)

This is a linear equation.

I.F

Multiply both sides of equation (4) by we get





Integrating both sides we get















which is the solution.

**Problem-02:** Solve the differential equation .

**Solution:** The differential equation is,

 … … … (1)

This is a Bernoulli’s equation.

Dividing the equation (1) by we get

… … … (2)

put



Now the equation (2) becomes,



… … … (3)

This is a linear equation.

I.F

Multiply both sides of equation (4) by we get





Integrating both sides we get









which is the required solution.

**Problem-03:** Solve the differential equation .

**Solution:** The differential equation is,

 … … … (1)

Equation (1) can be written as,

… … … (2)

This is a Bernoulli’s equation.

Dividing the equation (2) by we get

… … … (3)

put



Now the equation (3) becomes,



 … … … (4)

This is a linear equation.

I.F

Multiply both sides of equation (4) by we get





Integrating both sides we get











which is the required solution.

**Problem-04:** Solve the differential equation .

**Solution:** The differential equation is,

 … … … (1)

The equation (1) can be written as,



 … … … (2)

This is a Bernoulli’s equation.

Dividing the equation (2) by we get

… … … (3)

put



Now the equation (2) becomes,



 … … … (4)

This is a linear equation.

I.F

Multiply both sides of equation (4) by we get





Integrating both sides we get









which is the required solution.

**Problem-05:** Solve the differential equation .

**Solution:** The differential equation is,

 … … … (1)

This is a Bernoulli’s equation.

put



Now the equation (1) becomes,

… … … (2)

This is a linear equation.

I.F

Multiply both sides of equation (2) by we get





Integrating both sides we get













.

which is the required solution.

**Exercise:**

1. ****
2. ****
3. ****
4. ****
5. ****
6. ****

**Exact Differential Equations:** A differential equation is said to be an exactdifferential equationif it satisfies the following condition



**Working Rule:**

1. Integrate *M* with respect to *x* keeping *y* as constant,
2. Find out those terms in *N* which are free from *x* and integrate them with respect to *y*,
3. Add the two expressions so obtained and equate the sum to an arbitrary constant.

**Problem-01:** Solve .

**Solution:** Given that,

 … … … (1)

where,  and 





since, so the equation (1) is an exact differential equation.

Integrating *M* with respect to *x* we get



In *N*, terms free from *x* are whose integral with respect to *y* is



Therefore the general solution is

.

**Problem-02:** Solve .

**Solution:** Given that,

 … … … (1)

where,  and 





since, so the equation (1) is an exact differential equation.

Integrating *M* with respect to *x* we get



In *N*, terms free from *x* iswhose integral with respect to *y* is



Therefore the general solution is

.

**Problem-03:** Solve .

**Solution:** Given that,

 … … … (1)

where,  and 





since, so the equation (1) is an exact differential equation.

Integrating *M* with respect to *x* we get



In *N*, terms free from *x* are whose integral with respect to *y* is



Therefore the general solution is

.

**Problem-04:** Solve .

**Solution:** Given that,

 … … … (1)

where,  and 





since, so the equation (1) is an exact differential equation.

Integrating *M* with respect to *x* we get



In *N*, term free from *x* iswhose integral with respect to *y* is



Therefore the general solution is

.

**Exercise:**

1. 
2. 
3. 
4. 

**Equations reducible to exact differential equation:** A differential equation is not an exactdifferential equationif

.

But it can be reduced to an exact differential equation by multiplying a function of *x* and *y*, which is called an **integrating factor**.

**Rules for finding integrating factor:**Let the differential equation is,

 ... … … (1)

1. If then the integrating factor is.
2. If then the integrating factor is.
3. If *M* and *N* are both homogeneous function in *x*, *y* of degree *n*, then the integrating factor is .
4. If the equation (1) is of the form,then the integrating factor is 

**NOTE: 1.** If , thenand the equation reduces to ,

which can be easily solved.

**2.** If , thenand the equation reduces to ,

which can be easily solved.

**Problem-01:** Solve .

**Solution:** Given that,

 … … … (1)

where,  and 





since, so the equation (1) is not an exact differential equation.

However, 

Hence, I.F

Multiplying by I.F, the equation (1) becomes,

… … … (2)

which is exact now.

Let,  and 

Integrating  with respect to *x* we get



In , there is no term free from *x*.

Therefore the general solution is

.

**Problem-02:** Solve .

**Solution:** Given that,

 … … … (1)

where,  and 





since, so the equation (1) is not an exact differential equation.

However, 

Hence, I.F

Multiplying by I.F, the equation (1) becomes,

… … … (2)

which is exact now.

Let,  and 

Integrating  with respect to *x* we get



In , there is no term free from *x*.

Therefore the general solution is

.

**Problem-03:** Solve .

**Solution:** Given that,

 … … … (1)

where, and 





since, so the equation (1) is not an exact differential equation.

However, 

Hence, I.F

Multiplying by I.F, the equation (1) becomes,

… … … (2)

which is exact now.

Let,  and 

Integrating  with respect to *x* we get



In , the term free from *x*is *y* and integrating it with respect to y we have



Therefore the general solution is

.

**Problem-04:** Solve .

**Solution:** Given that,

 … … … (1)

where,  and 





since, so the equation (1) is not an exact differential equation.

But the equation (1) is a homogeneous differential equation.

Hence, I.F

Multiplying by I.F, the equation (1) becomes,

 … … … (2)

which is exact now.

Let,  and 

Integrating  with respect to *x* we get



In , there is no term free from *x*.

Therefore the general solution is

.

**Problem-05:** Solve .

**Solution:** Given that,

 … … … (1)

where,  and 





since, so the equation (1) is not an exact differential equation.

But the equation (1) is of the form,.

Hence, I.F

Multiplying by I.F, the equation (1) becomes,

 … … … (2)

which is exact now.

Let,  and 

Integrating  with respect to *x* we get



In , term free from *x*is , whose integral with respect to *y* is,



Therefore the general solution is









 .

**Exercise:**

1. 
2. 
3. 
4. 